

The QCD Phase Transition Region with Domain Wall Quarks

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Abstract. Results will be presented from a study of the QCD transition region using 2+1 flavors of fermions and a dislocation suppressing gauge action on a lattice with temporal extent of 8 and spatial extent 16 ($1.9 - 2.7$ fm). A series of temperatures from 140 through 200 MeV, separated by 10 MeV have been studied. All the simulations lie on a line of constant physics with 200 MeV pions, realized using domain wall fermion, a chirally symmetric fermion formulation. The chiral condensates, susceptibility, anomalous symmetry breaking and a detailed study of the Dirac spectrum will be described and compared with earlier staggered results.

Keywords: QCD Transition, Domain Wall Fermions, Dirac Spectrum

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The chiral phase transition of strongly-interacting matter has been an intriguing topic for decades. Many significant efforts, both theoretical and experimental, have been dedicated to understanding this phenomena. Lattice Quantum Chromodynamics, since the day of its birth, has offered a powerful technique for an *ab initio*, non-perturbative study of the transition region. Here we present a preliminary study of the transition region using a chiral formulation of lattice fermions in order to more accurately realize the chiral symmetry responsible for the transition.

GAUGE AND FERMION ACTIONS

While the gauge field can be implemented on the lattice in a straightforward manner, the usual lattice fermion schemes suffer from problems such as breaking of chiral symmetry or creating doublers *etc.*, which interfere with the exploration of the QCD chiral phase transition using lattice methods. The domain wall fermion (DWF) formulation [1, 2], a variant of Wilson fermions, avoids these problems at the cost of introducing an extra, s dimension. The left and right-handed chiral states are bound to the four dimensional boundaries (walls) at the two ends of the 5-D volume. Unphysical, massive degrees of freedom from the fifth dimension are to a large extent canceled by the introduction of a Pauli-Villars term in the simulation. The low modes produce a discrete, four-dimensional version of QCD.

The DWF formulation has chiral breaking symmetry under good control. The residual chiral symmetry breaking is characterized by a residual, additive quark mass: $m_{\text{res}}(L_s) = c_1 \exp(-\lambda_c L_s)/L_s + c_2/L_s$. If the extension in fifth dimension, L_s is taken to infinity, one will obtain the overlap fermion formulation which exactly preserves chiral symmetry. The infinite L_s limit commutes with the continuum extrapolation, allowing us to choose an appropriate L_s to circumvent the heavy computational burden of overlap fermions while keeping the chiral symmetry breaking minimal. Another benefit of the DWF formalism is that $U(1)_A$ symmetry is only broken by axial anomaly in contrast to staggered fermions, where the anomalous symmetry is also broken by a^2 artifacts.

In the simulation of QCD thermodynamics on a lattice with constant physical quark masses, one naturally needs a small bare quark mass. Since the residual mass is an additive correction to the bare quark mass in DWF formalism, *viz.* $m_q = m_{\text{res}} + m_{\text{input}}$, we must also minimize m_{res} . From expression for m_{res} , the residual mass can be decomposed into two parts. The first part is produced by standard five dimensional states and is exponentially suppressed; the other, comes from gauge field dislocations associated with changing topology and is only suppressed by $1/L_s$. Therefore, naively increasing L_s will not only be computationally expensive but also inefficient. Fortunately, the topology change associated with the second term also leads to zero modes of an unphysical 4-dimensional Dirac operator, $D_W(-M_5)$ so we could multiply a factor (*e.g.* determinant of that Dirac operator) to suppress those configurations. Since such a

factor will freeze topological tunneling, we add a twisted mass term to the determinant and introduce the ratio:

$$\mathcal{W}(M_0, \varepsilon_b, \varepsilon_f) = \frac{\det \left[D_W^\dagger(-M_0) D_W(-M_0) + \varepsilon_f^2 \right]}{\det \left[D_W^\dagger(-M_0) D_W(-M_0) + \varepsilon_b^2 \right]}. \quad (1)$$

This is the dislocation suppression determinant ratio (DSDR) [3, 4, 5]. This alters predominately the ultraviolet part of the theory, so the physical quantities in which we are interested are minimally affected.

TABLE 1. Summary of the ensembles.

T (MeV)	β	L_s	$m_{\text{res}}a$	$m_l a$	$m_s a$	$\langle \bar{\psi} \psi \rangle_l / T^3$	$\Delta \bar{\psi} \psi / T^3$	$\chi_{l, \text{disc.}} / T^2$
140	1.633	48	0.00612	-0.00136	0.0519	6.26(12)	7.74(12)	36(3)
150	1.671	48	0.00296	0.00173	0.0500	6.32(29)	6.10(29)	41(2)
150	1.671	32	0.00648	-0.00189	0.0464	8.39(10)	7.06(10)	44(3)
160	1.707	32	0.00377	0.000551	0.0449	5.25(17)	4.83(17)	43(4)
170	1.740	32	0.00209	0.00175	0.0427	4.03(18)	2.78(18)	35(5)
180	1.771	32	0.00132	0.00232	0.0403	3.16(15)	1.56(15)	25(4)
190	1.801	32	0.00076	0.00258	0.0379	2.44(9)	0.71(9)	11(4)
200	1.829	32	0.00046	0.00265	0.0357	2.19(8)	0.47(8)	10(3)

Table 1 summarizes the ensembles in our simulation. We use $2 + 1$ flavors of DWF with Iwasaki and DSDR gauge action. The spatial extent of the lattice is 16 (about 1.9 - 2.7 fm). The temporal extent is fixed to be 8 and the β values are chosen so that we have a range of temperatures from 140 to 200 MeV. The bare quark masses are also adjusted to make our ensembles lie on a line of constant physics with a pion mass of about 200 MeV.

CHIRAL SYMMETRY

The first quantity of interest is the chiral condensate shown as a function of Monte Carlo time in fig. 1. As compared to those below 160 MeV, the evolutions of the chiral condensate for temperatures above 160 MeV display a distinct character, fluctuating above a base line. The graph of the disconnected susceptibilities gives a more direct indication of a transition temperature around 160 MeV, which agrees quite well with the staggered results. However, since our spatial volume is quite limited (an aspect ratio 2), a quantitative comparison is likely premature.

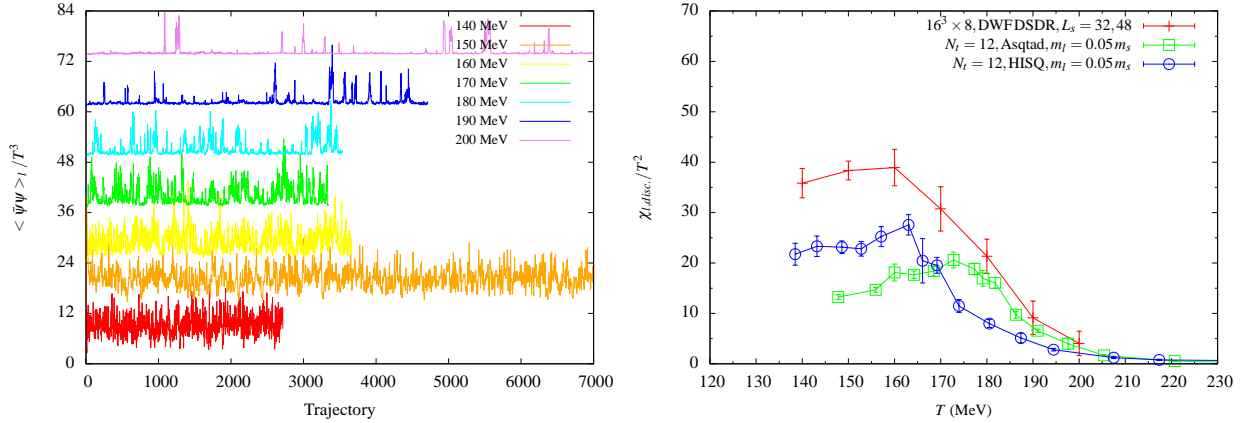


FIGURE 1. Evolution of $\langle \bar{\psi} \psi \rangle_l$, each shifted up by 12 units (left) and preliminary results of the disconnected susceptibility, normalized in a consistent scheme differing from that used in Table 1, at various temperatures (right).

The chiral condensate is closely related to the small eigenvalues of the Dirac operator, which thus provides an alternative perspective into the symmetries associated with the QCD phase transition. For instance, the well-known Banks-Casher relation [6], $\lim_{m_l \rightarrow 0} \lim_{V \rightarrow \infty} \langle \bar{\psi} \psi \rangle_l = -\pi \lim_{\lambda \rightarrow 0} \lim_{m_l \rightarrow 0} \lim_{V \rightarrow \infty} \rho(\lambda)$, relates the chiral condensate and the $\lambda = 0$ value of the density $\rho(\lambda)$ of Dirac eigenvalues.

The Dirac spectrum also provides unique insights into the anomalous $U(1)_A$ symmetry. Specifically, the $U(1)_A$ -breaking difference of pseudoscalar and scalar susceptibilities is given by:

$$\Delta_{\pi-\delta} \equiv \frac{\chi_{\pi} - \chi_{\delta}}{T^2} = \int d\lambda \rho(\lambda) \frac{4m_l^2}{(m_l^2 + \lambda^2)^2}. \quad (2)$$

While $\Delta_{\pi-\delta}$ is expected to receive a small, dilute instanton gas contribution from $\rho(\lambda) \propto m^2 \delta(\lambda)$ at large temperatures, there may also be larger, lower-temperature non-semi-classical contribution from $\rho(\lambda) \propto \lambda$. We used Kalkreuter-Simma's method [7] to calculate the lowest 100 eigenvalues of the hermitian version of DWF Dirac operator. The eigen-spectrum is then renormalized using an method similar to Giusti and Luscher [8].

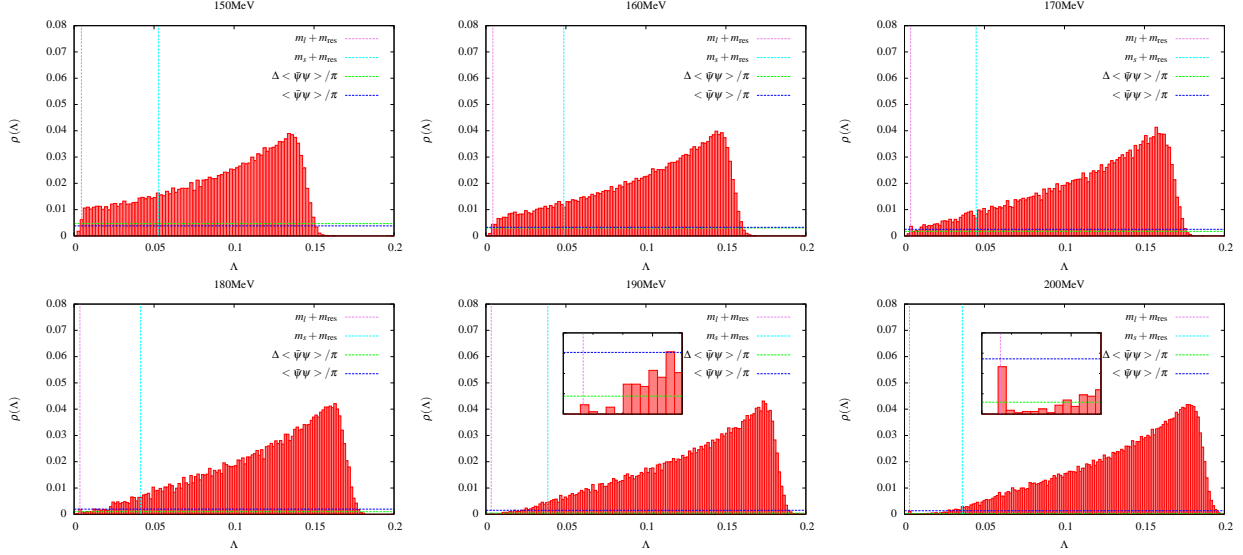


FIGURE 2. Dirac spectrum of $T = 150 - 200$ MeV ensembles.

As can be seen in fig. 2, the renormalization aligns the light quark mass (dashed vertical line) with the near-zero mode peak. It also compensates for the discrepancy in the Banks-Casher relation, which however still does not agree at about 50% level below the transition. We attribute the discrepancy to the finite volume or finite mass effects. Above 170 MeV, the chiral condensate and eigendensity both start to vanish as expected for temperatures above the transition. But it remains unclear whether the slope of the eigendensity vanishes at 180 MeV. At even higher temperature, a possible gap starts to emerge, indicating an effective restoration of $U(1)_A$ symmetry.

We conclude that DWF provides a well-suited tool to study the QCD phase transition. From the chiral susceptibility, we find indication of a transition temperature around 160 MeV. Well above the transition, we observe possible signals of $U(1)_A$ restoration. Larger volume results and chiral extrapolation will be needed for a more definite conclusion.

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